Journal of Engineering Physics and Thermophysics, Vol. 65, No. 1, 1993

MATHEMATICAL MODELING OF SHS EXTRUSION. 2. RHEODYNAMIC MODELS

L. S. Stel'makh and A. M. Stolin

UDC 532.135

A review of results of mathematical modeling of the rheodynamics of SHS hot compaction and extrusion of combustion products is presented.

Rheodynamic Models. Earlier, in the thermal models [1] the role of the rheological factor was included in a simplified way by using an effective parameter such as the material viability temperature. Further complication of the model was done by including high-temperature deformation and compaction of combustion products. Rheodynamic models were formulated that revealed directly the role of volume and shear viscosities and their dependences on temperature and density. In that stage of mathematical modeling its important elements were analytical solutions that included the relations of hot compaction and extrusion of powder materials and analysis of extreme conditions of extrusion. Numerical analysis of the influence of temperature nonuniformity and heat transfer conditions on the relations of combustion product extrusion is carried out.

The Rheological Model. It should be noted that the object of the study was powders of refractory compounds. At high temperatures they behave as highly viscous liquids [2-4]. It is assumed that compaction of porous materials occurs due to the outward flow of the incompressible basic material to the void. The course of change of the shape and volume depends on the presence of two material characteristics, namely, volume and shear viscosities. For compressible materials of powdered refractory compounds investigation of their compaction ability is of primary importance. Therefore, volume viscosity and its dependence on density and temperature play one of the leading roles. In the theory of hot compaction of powdered materials a purely continuum approach is adopted [2, 3], which consists in defining the volume ξ and shear μ viscosities as a function of instantaneous porosity:

$$\mu = \eta_{com} f_1(\rho),\ \xi = \eta_{com} f_2(\rho),$$

where η_{com} is the viscosity of the solid substrate, which can depend on the deformation and temperature parameters, and $f_1(\rho)$ and $f_2(\rho)$ are functions of the density specified by analytical approximation of empirical relations. Expressed in such a form, the viscosity of the solid substrate functions as a material constant corresponding to the instantaneous porosity that depends on the temperature and deformation parameters. It should be noted that similar considerations are used in the determination of various effective transfer coefficients in inhomogeneous media.

For compressible materials a main experimental objective is to investigate the dependences of shear and volume viscosities on the deformation parameters and instantaneous porosity. In this connection, the problem of finding a method of measuring the main rheological properties and devising suitable equipment is being solved [5]. Development of the theory of viscosimetric flows is under way now, and a description of compaction and extrusion of these materials is given, as applied to particular technological conditions [1]. In essence, a new research trend - high-temperature rheology of powder materials - is being created [12].

Analytical Models of Compaction and Extrusion of Porous Materials. In viscometric theory it is necessary to choose idealized flow types which are accessible for experimental realization and which can be described theoretically with the aid of analytical methods. Here solutions of very simple problems on compaction and extrusion of a porous material under the action of the hydrostatic component of the stress tensor are ordinarily used. The flow types chosen

---

Institute for Structural Macrokinetics, Russian Academy of Sciences, Chernogolovka, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 65, No. 1, pp. 90-94, July, 1993. Original article submitted January 29, 1992.

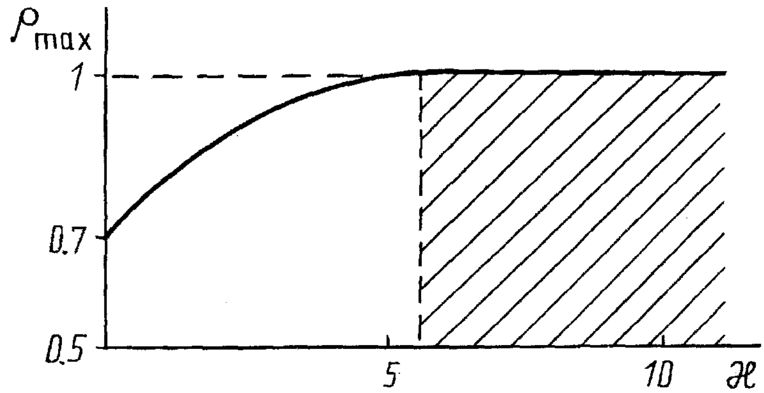

Fig. 1. Plot of the maximum density ρ_{max} vs the parameter κ.

are characterized by the presence of just one velocity component $V_i \neq 0$ and one derivative $\partial V/\partial X_i \neq 0$. In this case the invariants of the velocity and deformation tensors

$$J_1 = \frac{\partial V_i}{\partial X_i}, \quad J_2 = \left(\frac{\partial V_i}{\partial X_i}\right)^2, \quad J_3 = \left(\frac{\partial V_i}{\partial X_i}\right)^3,$$

and any dependence of the viscosities ξ and μ on J_{com} only lead to the dependence on $\partial V_i/\partial X_i$. It should be noted that the stress state of the materials is characterized here by three nonzero normal stress components. Among such kinds of flows we can mention cases of unilateral axial compression of material in a cylindrical mold and radial compression of material comfined between two boundaries (cylinders and spheres), one of which is fixed and the other displaced in the radial direction.

Finding the dependence of the macroscopic density of a porous material on pressure is the main objective of theoretical solution of such problems. Processes of compaction and extrusion of a porous material subjected to unilateral compression in a cylindrical mold has been described most efficiently. The type of flow chosen is described by a set of continuity equations together with the rheological relations reported in [13]: equilibrium equations

$$\frac{\partial \sigma_{zz}}{\partial z} = 0, \tag{1}$$

boundary and initial conditions

$$\sigma_{zz}\big|_{z=H(t)} = -P, \quad \rho(z, t)\big|_{t=0} = \rho_0(z). \tag{2}$$

The shear μ and volume ξ viscosities are defined by the following porosity functions:

$$\mu(\rho) = \eta_{com}\rho^m, \quad \xi(\rho) = \eta_{com}\frac{\rho^\alpha}{1-\rho}. \tag{3}$$

The exponents vary in the following ranges: $1 \le \alpha \le 4$, $0 \le m \le 3$, $0.5 \le \rho \le 1$.

Various compaction regimes (regular, wave, and transient) were revealed and criteria for their realization were found [8, 9].

The analytical solutions obtained were used for analysis of limiting extrusion conditions for materials without compaction and for material compaction followed by extrusion [10, 11]. The main dimensionless parameter controlling the compaction and extrusion process is $\kappa = t_{ex}/t_*$, which characterizes the ratio of the extrusion $t_{ex} = q_0/\overline{P}$, $\overline{P} = KP^nS_1/S_0\rho_1$, and compaction $t_* = 4\mu_1/3P$ times. It can be seen from Fig. 1 that at small κ ($\kappa << 1$) an uncompacted specimen is extruded ($\rho_{max} < 0.85$), whereas at $\kappa \ge 5$ the material is compacted and then extruded. In essence, this is the answer to the question about the feasibility of stage-by-stage analysis of the process. Strictly speaking, compaction and extrusion of compressible powder materials occur in parallel. Extrusion is not always a step-by-step

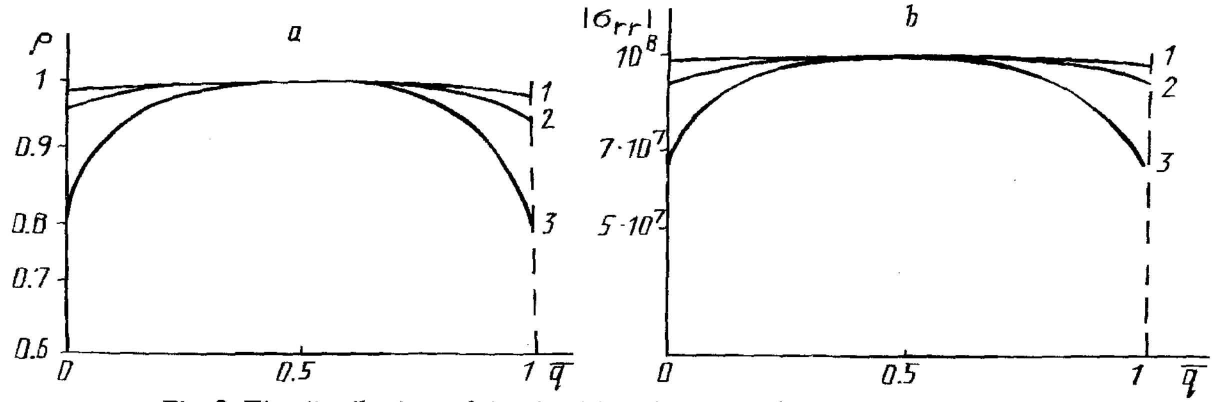

Fig. 2. The distributions of density (a) and stresses (b) over the specimen with various lag times t_{lg}: 1) $t_{lg} = 0$; 2) 5 sec; 3) 10 sec. (σ_{rr}), Pa.

process. In practice this favorable situation should be provided using various means. In particular, this situation is realized for a narrow slit with a large hydraulic resistance.

The analytical solutions obtained are an element of mathematical modeling, since they allow qualitative consideration of the relations of the extrusion process. So, in compaction in a cylindrical mold under adiabatic conditions density is self-leveled. The effect of density nonuniformity on extrusion is different. With the same density difference $\Delta\rho$ ($\Delta\rho = \rho_{max} - \rho_{min}$) and initial distribution but with different positions of the pellet (direct and inverse), the density distributions in the extruded rod are different [10]. More favorable is the case where "the bottom is denser than the top" (the density nonuniformity in the rod decreases). The inverse position results in a substantial increase in the density inhomogeneity (as much as twice).

Nonisothermal Models for Compaction and Extrusion. At present there are thermodynamic models [6] that contain both the thermal model equations (1)-(3) of [1] and the rheodynamic model equations (1) to (3). Moreover, additional relations are introduced that take account of the dependence of liquid properties on temperature. These models were successfully used to describe such physical regimes as material undercompaction of a material and to find the region of optimal parameters for production of compact materials. For example, in Fig. 2 the effect of the time lag (the time from the initiation of the chemical reaction to pressure application) on the density and stress distribution in a material is shown. One can see that as the time lag increases, due to rapid material cooling, the radial stress drop along the specimen rises, and a larger part of the material remains undercompressed and the limiting material density also decreases substantially.

Analysis of numerical results shows that the boundary curve, below which the operation region lies, is the necessary condition for the existence of the compaction regime [6].

Conclusion. In this article, problems of the description of deformation and a nonuniform space-time temperature distribution are isolated from a wide range of problems of mathematical modeling. Experience in using the thermal and rheodynamic models in the commercial production of very long articles designed for various purposes (heating elements, electrodes for electric spark alloying etc.) permits a conclusion to be drawn about their effectiveness and usefulness for investigation of the production process, forecasting, and development of recommendations and for qualitative explanations of the problem situations. At present successive activity is under way aimed at using a set of programs developed for computerization of commercial SHS compacting.

However, some interesting and important problems have not been solved as yet. Further development of rheodynamic models will presumably take place along the line of including the nonlinearity of rheological properties, and of great importance is information about the rheological behavior of refractory powder materials at high temperatures. Material synthesis and article shaping are accompanied by some nonequilibrium transformations, chemical (through formation of intermediate reaction products), phase (crystallization), and structural (dispersion of solid reactant particles). The transformations essentially change the rheological and thermal properties of the materials and influence the operation regimes and the quality of products. Inclusion of these factors in their complex interaction is a future trend in development of mathematical modeling of SHS extrusion.

NOTATION

t, time; r, z, transverse and longitudinal coordinates; ρ, relative density of materials; ρ_1, η_{com}, density and viscosity of the incompressible base of a material; μ, ξ, shear and volume viscosities of a material; σ_{rr}, σ_{zz}, radial and axial stresses; S_0, S_1, cross-sectional areas of the chamber and the pass; P, pressure on the press plunger; q_0, relative mass of the blank; κ, ratio of characteristic compaction to extrusion times; V, material flow velocity; ρ_{max}, maximum relative density; $\rho_0(q)$, volume distribution of density of the initial blank .

REFERENCES

1. L. S. Stel'makh, A. M. Stolin, and A. G. Merzhanov, Inzh.-Fiz. Zh., **65**, No. 1, 83-89 (1993).
2. M. S. Kovalchenko, Theoretical Fundamentals of High-Temperature Treatment of Porous Materials by Pressure [in Russian], Kiev (1972).
3. V. V. Skorokhod, Rheological Fundamentals of Sintering Theory [in Russian], Kiev (1972).
4. Ya. E. Geguzin, Why and How Does the Void Disappear? [in Russian], Moscow, Nauka (1976).
5. L. M. Buchatskii and A. M. Stolin, Inzh.-Fiz. Zh., **49**, No. 4, 644-652 (1989).
6. L. S. Stel'makh, N. N. Zhilyaeva, and A. M. Stolin, Inzh.-Fiz. Zh., **61**, No. 1, 33-40 (1991).
7. L. M. Buchatskii, A. M. Stolin, and S. I. Khudyaev, Poroshk. Metall., No. 12, 9-14 (1989).
8. A. M. Stolin, N. N. Zhilyaeva, and B. M. Khusid, Inzh.-Fiz. Zh., **59**, No. 2, 248-254 (1991).
9. N. N. Zhilyaeva and A. M. Stolin, Inzh.-Fiz. Zh., **59**, No. 6, 984-988 (1990).
10. L. S. Stel'makh, A. M. Stolin, and B. M. Khusid, Inzh.-Fiz. Zh., **61**, No. 2, 268-276 (1991).
11. L. S. Stel'makh and A. M. Stolin, Dokl. Ross. Akad. Nauk, **322**, No. 4, 732-736 (1992).
12. A. M. Stolin, in: Rheophysics and Thermal Physics of Nonequilibrium Systems, Coll. Mat. Int. Workshop, Pt. 2, Minsk (1992), pp. 11-19.
13. L. S. Stel'makh, N. N. Zhilyaeva, and A. M. Stolin, Inzh.-Fiz. Zh., **63**, No. 5, 612-622 (1992).